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ATTENUATION, DISPERSION, AND THE WAVE GUIDE OF THE G WAVE

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ABSTRACT

Using the strain seismograms of the New Guinea earthquake of 1938 and the Kamchatka earthquake of 1952, the decrement of the G wave in the mantle of the earth was determined from the comparison of the amplitude of Fourier components, which are obtained by analyzing the G phases at different epicentral distances. The value of $1/Q$ thus obtained is a little larger than that given by M. Ewing and F. Press using mantle Rayleigh waves, but is not much different. The phase velocity was also calculated using the argument of the Fourier transform. The dispersion curves obtained from (G_1 and G_3), (G_2 and G_4) of the New Guinea earthquake and (G_1 and G_3) of the Kamchatka earthquake agree quite well, giving a nearly constant group velocity 4.4 km/sec. as was anticipated. Theoretical consideration of the distribution of shear velocity that serves as the wave channel for the guidance of the G wave was given, and the shear velocity was calculated applying the method of T. Takahashi to the dispersion curve derived from the condition of constant group velocity, which is a direct consequence of the fact that the G wave shows almost no dispersion. The $V_s(z)/V_0$ curve which was derived theoretically agrees well with the curve given by the distribution of shear velocity of Jeffreys-Bullen in the range between one and several hundred kilometers.

1. Introduction

For many years the existence of a special type of wave with a large amplitude, long period, and transverse polarization has been noticed. This wave is observed only in seismograms of severe earthquakes[1] and the propagation velocity is about 4.4 km/sec., which is determined fairly well because the wave is pulse-like and the phase consists of only a single oscillation or two.

Today the G wave, named after B. Gutenberg,[2] who first observed it, attracts keen interest because of its peculiar properties. The author has attempted analysis of this phase using seismograms which were kindly given by Professor H. Benioff.

The contents are classified into three parts:

a) Determination of the decrement of waves from the comparison of the amplitude of Fourier components, which are obtained by analyzing the observations at different epicentral distances.

b) Calculation of the phase velocity using the argument of the Fourier transform.

c) Theoretical consideration of the distribution of shear velocity that serves as the wave channel for the guidance of the G wave.

Although we have only the observation made by the strain seismograph at Pasadena, we can detect not only the G phase that traveled the shortest path but also those which followed the major arc and those which circled the earth; these provide enough data for the determination of the absorption of the earth's mantle.

Manuscript received for publication January 27, 1958.

[1] B. Gutenberg and C. F. Richter, "On Seismic Waves," *Gerlands Beitr. z. Geophys.*, 43, Heft 1/2 (1934); Dominico Di Fillipo, "L'onda 'G' di Gutenberg nel terremoto della Azorre del 25 novembre 1941," *Annali di Geofisica*, 3, No. 1 (1950); A. Imamura, *Theoretical and Practical Seismology* (Tokyo: Maruzen); M. Ewing and F. Press, "Surface Waves and Guided Waves," *Encyclopedia of Physics*, XVII, Geophysics I, p. 125.

[2] *Loc. cit.* (n. 1, *supra*).

The method of calculating phase velocity having already been developed,[3] the newly suggested formula of Takahashi[4] was applied to the computation of shear velocity within the mantle, and worked well.

2. Attenuation of G wave

As was pointed out above, the properties of the G wave suggest that it is a kind of guided wave. Therefore the energy of the wave is, with regard to space, restricted to the part not far from the surface of the earth. Also it is pulse-like and shows no dispersion; consequently the energy is, with regard to time as well, restricted within a certain relatively short time interval.

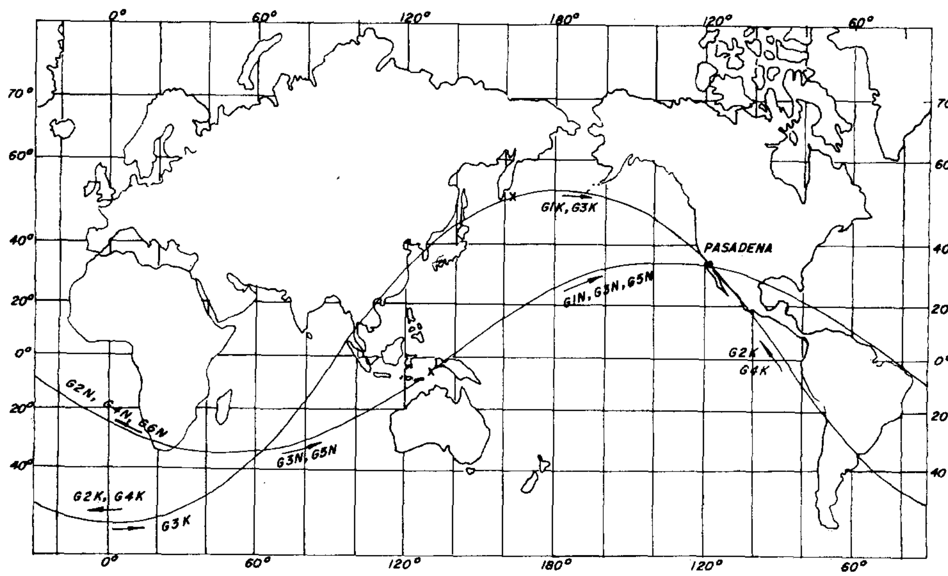


Fig. 1. Epicenters of New Guinea (1938) and Kamchatka (1952) earthquakes and paths of G wave. G1N implies the G_1 wave of the New Guinea earthquake, and so on.

In general an arbitrary function with a limited total fluctuation is expressed in the following form, which is Fourier's double integral theorem.

$$f(t;\,\Delta) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{\infty} f^*(p;\,\Delta) \exp\,(ipt)\,dp$$

and (2.1)

$$f^*(p;\,\Delta) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{\infty} f(t;\,\Delta) \exp\,(-ipt)\,dt$$

[3] Paolo E. Valle, "Sulla misura della velocità di gruppo delle onde sismiche superficiali," *Annali di Geofisica*, 2, No. 3 (1949); Y. Satô, "Analysis of Dispersed Surface Waves, I, II, and III," *Bull. Earthq. Res. Inst., Tokyo Univ.*, 33, Pt. 1 (1955); 34, Pt. 1 and Pt. 2 (1956).
Dr. P. E. Valle kindly informed the author of the existence of his paper published several years ago. The principal part of the thoery of the method using Fourier transform adopted in the present paper, which is a development of the work undertaken a few years ago, is covered by his study. The author's sincere thanks are due to Dr. P. E. Valle. The theory is described here again only for the sake of convenience, using unified notations.

[4] T. Takahashi, "Analysis of Dispersion Curves of Love Waves," *Bull. Earthq. Res. Inst., Tokyo Univ.*, 33, Pt. 3 (1955).

where p is the frequency and Δ is the epicentral distance. The spectrum of the curve $f(t; \Delta)$ is $|f^*(p; \Delta)|$, which is given by the second expression of (2.1). Although the integration of the expression extends from $-\infty$ to ∞, the disturbance is, as is stated above, restricted within a short range of time and $f(t; \Delta)$ has a value other than zero only in a limited domain (a, b). Therefore we have

$$f^*(p; \Delta) = \frac{1}{\sqrt{(2\pi)}} \int_a^b f(t; \Delta) \exp(-ipt)\, dt \tag{2.2}$$

2.1. Data:

The seismograms employed in the present study are those of the New Guinea earthquake of 1938 and the famous Kamchatka earthquake of 1952. Brief data concerning them are given in table 1.

TABLE 1

	New Guinea earthquake	Kamchatka earthquake
Date	Feb. 1, 1938	Nov. 4, 1952
Occurrence time (G.C.T.)	$19^h04^m21^s$	$16^h58^m20^s$
Epicenter	5.°0 S, 131½° E	52¾° N, 159½° E

2.2. New Guinea Earthquake:

The observation was made at Pasadena, California, and the epicentral distance was 109.°6. The wave path is illustrated in figure 1. The instrument is the Benioff linear strain seismograph. The component is NS, which is approximately perpendicular to the geodetic line connecting the epicenter and the station. The galvanometer has a period of 70 sec. in this case, and a part of the seismogram is reproduced in figure 2. We could observe even G_6 on the seismogram, wherefore we measured all of the six phases and made the calculation given by equation (2.2). In doing this computation, if we make the interval (a, b) too small, we may miss a part of the disturbance belonging to the G phase, while if we make the interval too large, taking the safe side, other phases or background noise may be involved in the interval and the spectrum will also be affected. Therefore we adopted various sizes of (a, b) and tried the computations so that we could see the effect cited above. The curve was measured at every 0.1 minute and the numerical computation was performed by an electronic computer. The result is given in figure 3. There is no significant difference between the values obtained by using various sizes of integration interval; therefore we may trust the result obtained above. However, these curves involve rapid and irregular fluctuations, which seem to have no special meanings. So we smoothed the curves, which are given in figure 4. These smoothed curves are used for the following study. The broken line indicates the part where the amplitude is too small to be reliable.

This figure shows many interesting things. First, G_1 is the largest and G_6 the smallest, which fact very naturally suggests the effect of absorption. However, there is an exceptional part in which G_2 is larger than G_1, a fact which never occurs without some special reason. Secondly, the maxima of the curves shift to the lower frequency side from G_1 to G_6. This implies that the oscillation with higher frequency is absorbed more rapidly than that with lower frequency. This is an interesting phenomenon and makes us try to determine the attenuation coefficient as a function of frequency or period. The amplitude decrement is given by the function

$$\exp(-\gamma\Delta), \text{ where } \gamma = \pi/QVT \qquad (V = \text{phase velocity}; T = \text{period}) \tag{2.3}$$

and the value of γ and Q can be obtained by comparing the amplitude of waves observed at different epicentral distances.

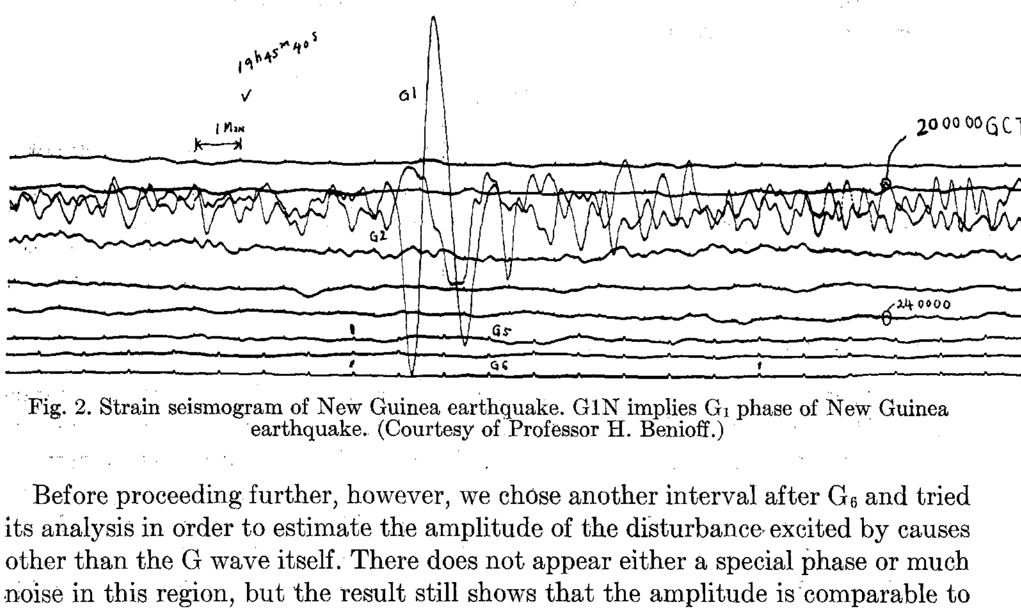


Fig. 2. Strain seismogram of New Guinea earthquake. G1N implies G_1 phase of New Guinea earthquake. (Courtesy of Professor H. Benioff.)

Before proceeding further, however, we chose another interval after G_6 and tried its analysis in order to estimate the amplitude of the disturbance excited by causes other than the G wave itself. There does not appear either a special phase or much noise in this region, but the result still shows that the amplitude is comparable to those of G_5 and G_6 (see fig. 5). Therefore we omitted these two phases for the determination of the attenuation coefficient. We only adopted the combinations (G_1-G_3) and (G_2-G_4) and did not try either of the combinations G_1 and G_2 or G_2 and G_3, because the waves propagated in the opposite directions seem to have different characteristics, as is easily suspected from the fact that there is a part where G_2 is larger than G_1.

γ is given in figure 6 and $1/Q$ in figure 7. The mean values of $1/Q$ are:

Period	360	216	108	72	54	43.2 (sec.)
$1/Q$	13½	8½	9	8	6	4 ($\times 10^{-3}$)

while the value given by M. Ewing and F. Press,[5] using mantle Rayleigh waves, is

$$1/Q = 665 \times 10^{-5} \quad \text{for } T = 215 \text{ sec.}$$
$$673 \times 10^{-5} \quad 140 \text{ sec.}$$

[5] M. Ewing and F. Press, "An Investigation of Mantle Rayleigh Waves," *Bull. Seism. Soc. Am.*, 44, No. 2A (1954).

2.3. Kamchatka Earthquake:

A similar procedure was carried out for the analysis of the seismogram of the Kamchatka earthquake of 1952. The observation was also made at Pasadena. The epicentral distance was 58°.6, and the galvanometer period in this case was 180 sec. The seismogram is shown in figure 8 and the spectra of the G-phases are in figure 9. Smoothed spectra are in figure 10, and γ is given in figure 11. Values of $1/Q$, a little larger than the previous ones, are also plotted in figure 7. They are

Period..........	360	216	108	72	(sec.)
$1/Q$.............	19	12	11	8	($\times 10^{-3}$).

3. Velocity of G wave

In the preceding section we used the absolute value of the Fourier transform and worked out the coefficient of absorption of the medium. Now we will calculate the velocity of propagation of the G wave by means of the argument of the transform. The method has been used before by the present author.[6]

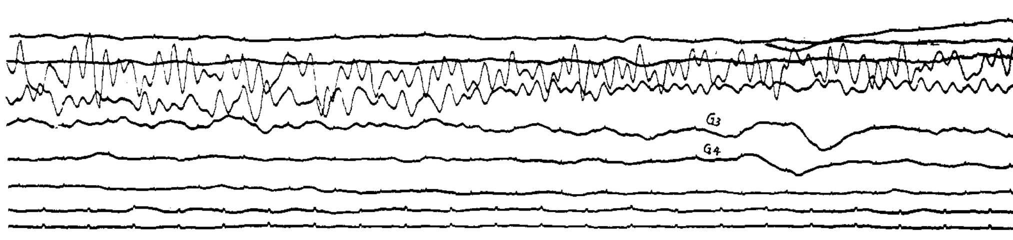


Fig. 2. *(Continued)*

Suppose there is a wave propagated two-dimensionally and the disturbance is given by $f(t;\,\Delta)$ (t = time; Δ = epicentral distance). Using the Fourier transform we have

$$f(t;\,\Delta_0) = \frac{1}{\sqrt{(2\pi)}}\int_{-\infty}^{\infty} f^*(p;\,\Delta_0)\exp(ipt)\,dp \tag{3.1}$$

At another point we have another disturbance expressed by the function

$$f(t;\,\Delta) = \frac{1}{\sqrt{(2\pi)}}\int_{-\infty}^{\infty} f^*(p;\,\Delta)\exp(ipt)\,dp \tag{3.2}$$

which can be also expressed by the following equation

$$f(t;\,\Delta) = \frac{1}{\sqrt{(2\pi)}}\int_{-\infty}^{\infty} R(\Delta_0,\,\Delta)f^*(p;\,\Delta_0)\exp\left\{ip\left(t - \frac{\Delta - \Delta_0}{V(p)}\right)\right\}dp \tag{3.3}$$

In this expression $R(\Delta_0,\,\Delta)$ is the factor that gives the decrease of amplitude on the way from $\Delta = \Delta_0$ to Δ, and $\exp\{-ip(\Delta - \Delta_0)/V(p)\}$ gives the phase shift. $V(p)$

[6] Y. Satô, *loc. cit.*, (n. 3, *supra*).

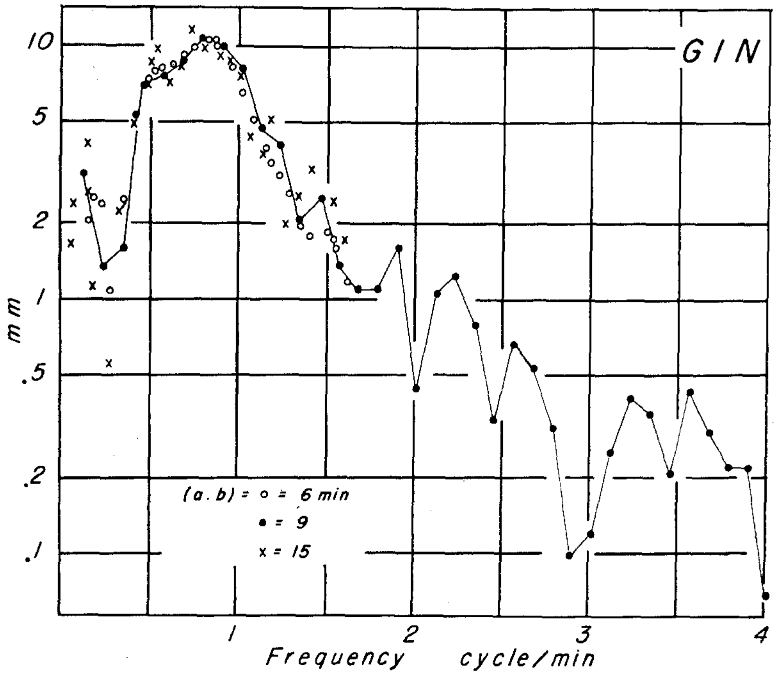


Fig. 3. Spectrum of G wave of New Guinea earthquake given in figure 2; (a, b) in each part of the figure is the range of integration. Cf. expression (2.2) in text.

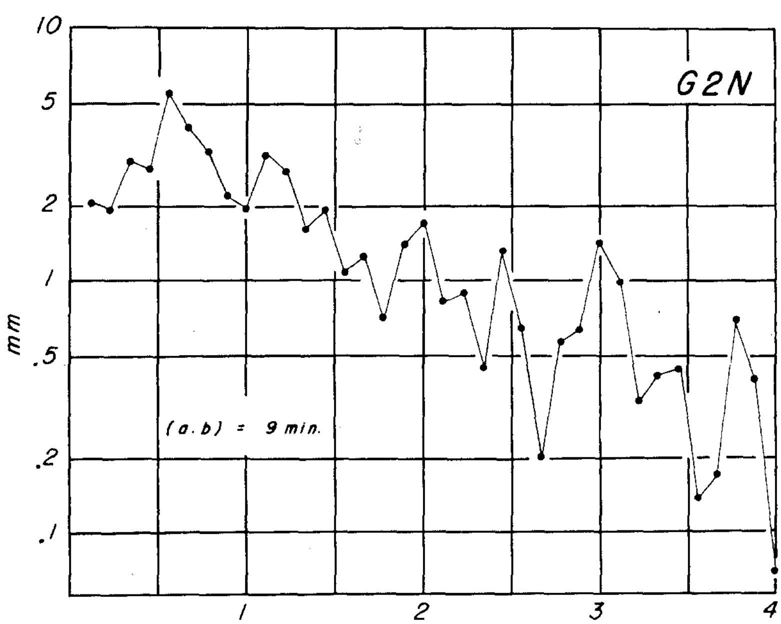
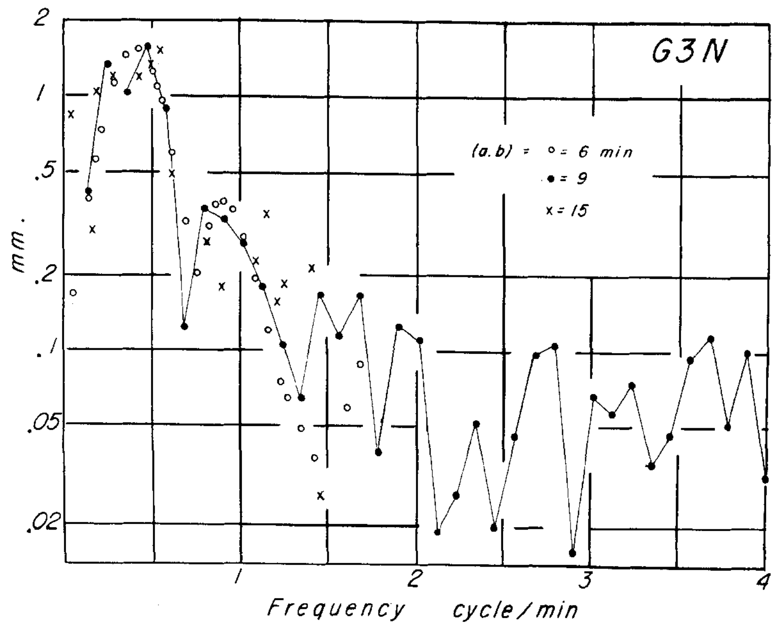


Fig. 3. (*Continued*)



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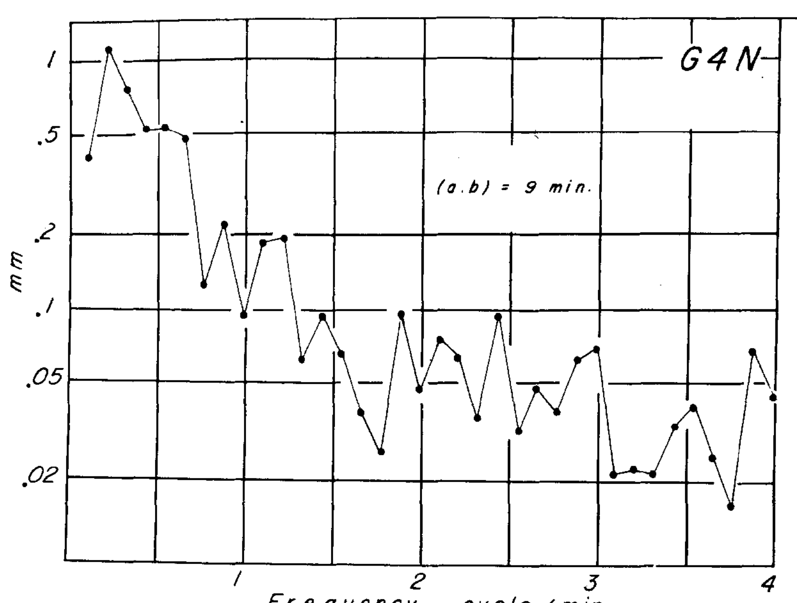

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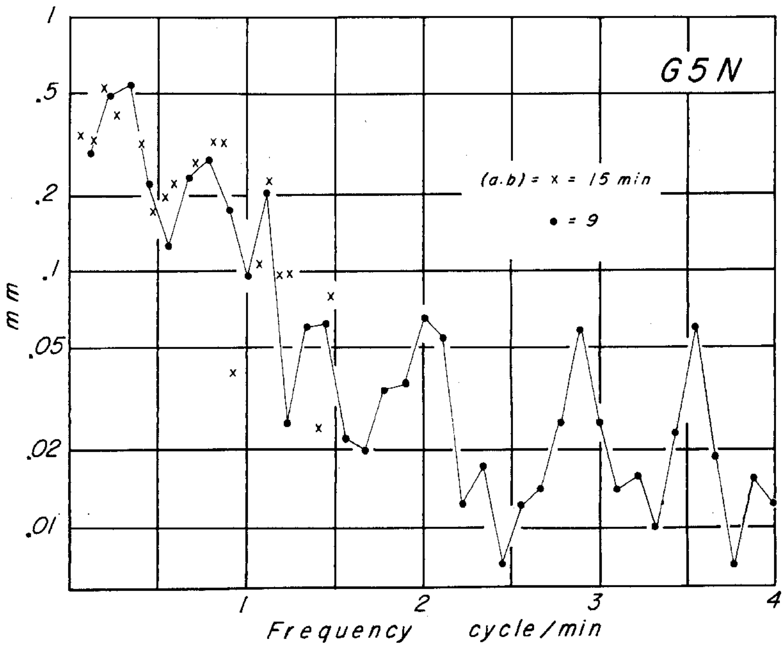

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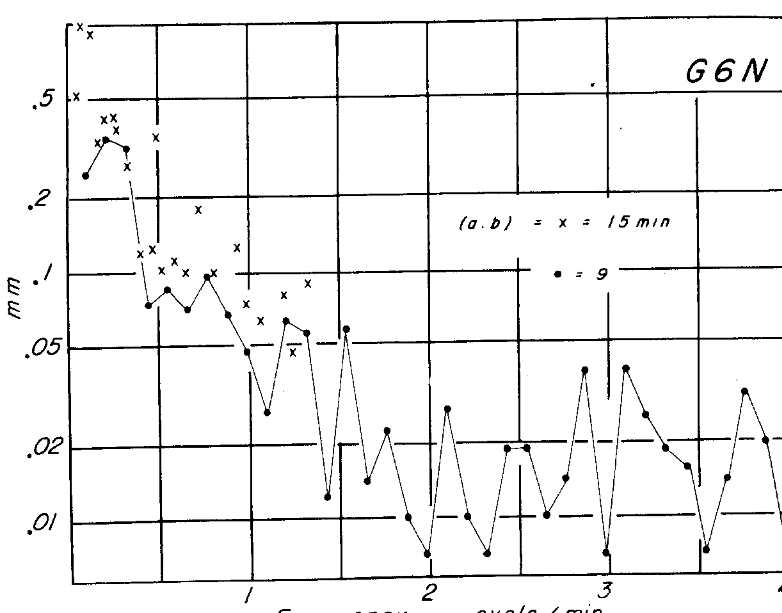

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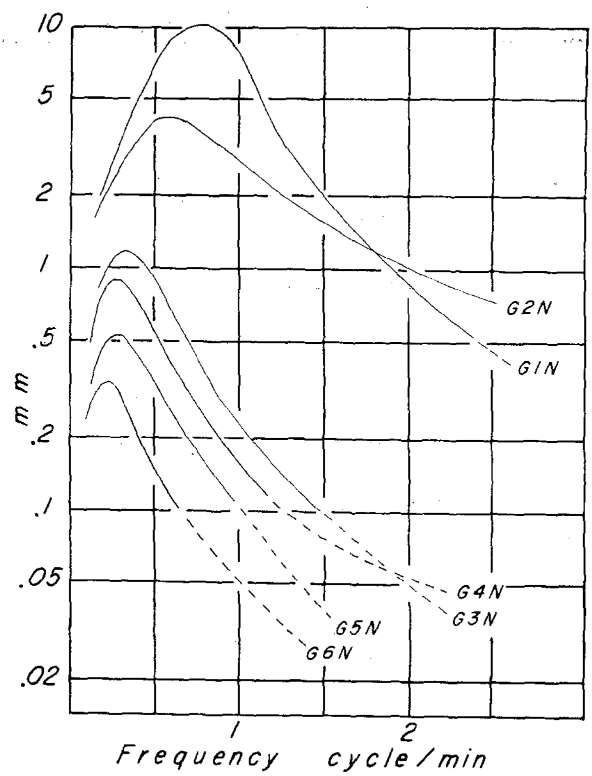


Fig. 4. Smoothed spectra of G phases of New Guinea earthquake.

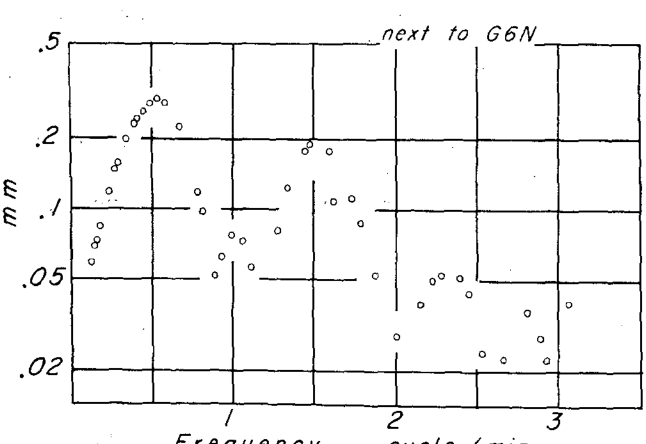


Fig. 5. Spectrum of the part of curve next to G_6 phase. Neither a special phase nor a strong noise seems to be present.

is the wave velocity with a frequency p. Equating the integrands in two expressions (3.2) and (3.3), and taking the argument, we can get a useful formula, namely,

$$\mathfrak{C}(p;\Delta) - \mathfrak{C}(p;\Delta_0) = p(\Delta - \Delta_0)/V(p) \tag{3.4}$$

where

$$f^*(p;\Delta) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{\infty} f(t;\Delta) \exp(-ipt)\, dt$$

$$= F(p;\Delta) \exp\{-i\mathfrak{C}(p;\Delta)\}$$

$F(p;\Delta)$ is a real function and $-\mathfrak{C}(p,\Delta)$ is the argument of $f^*(p,\Delta)$.

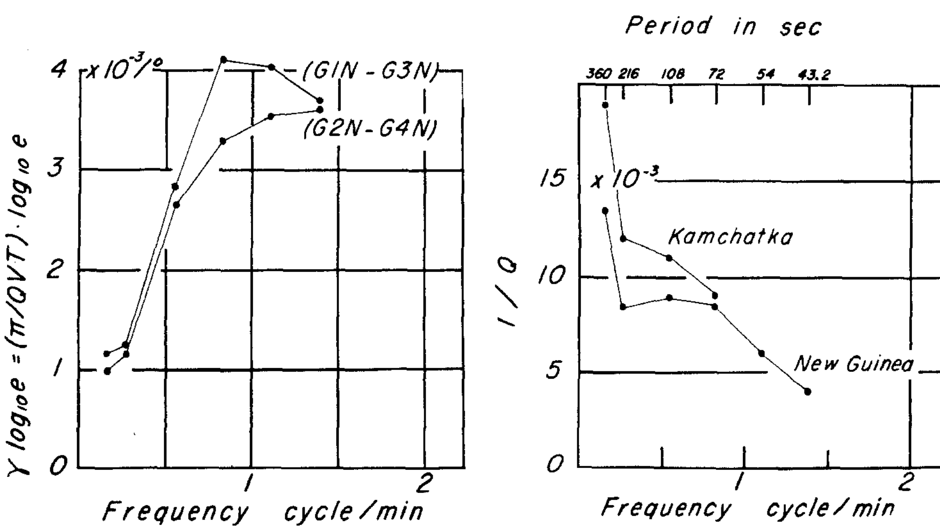


Fig. 6. (Left.) The value of decrement γ in the expression (2.3). V: phase velocity. T: period.

Fig. 7. (Right.) $1/Q$ as a function of the frequency or the period.

If two observations, say at $\Delta = \Delta_1$ and $\Delta = \Delta_2$, are available, we can solve the equation of the type (3.4) and obtain $\mathfrak{C}(p;\Delta_0)$ and $V(p)$ for any value of p. Neglecting a constant factor $R(\Delta_0, \Delta)$, the spectrum of the disturbance should always have the same shape.

Wave velocity is given by

$$V(p) = p(\Delta_2 - \Delta_1)/\{\mathfrak{C}(p;\Delta_2) - \mathfrak{C}(p;\Delta_1)\}$$

and

$$\mathfrak{C}(p;\Delta) = \arctan\left\{\int_{-\infty}^{\infty} f(t;\Delta)\sin pt\, dt \bigg/ \int_{-\infty}^{\infty} f(t;\Delta)\cos pt\, dt\right\} + 2n\pi \tag{3.5}$$

3.1. Effect of a seismograph:

In section 3 we dealt with true ground motion $f(t;\Delta)$. However, we usually use seismograms which have been more or less modified by seismographs; hence we must develop a theory if we want to use the seismogram itself without integrating and obtaining the true ground motion.

Suppose $fs(t; \Delta)$ is the signal recorded by a seismograph.

Two functions $fs(t; \Delta)$ and $f(t; \Delta)$ are connected by the following equation:

$$Ls[fs(t; \Delta)] = L[f(t; \Delta)] \tag{3.6}$$

where Ls and L are linear differential operators, and in the usual mechanical seismographs take the form

$$Ls[\quad] = P\frac{d^2}{dt^2} + Q\frac{d}{dt} + R$$

$$L[\quad] = -N\frac{d^2}{dt^2}$$

Even if the operators L and Ls take more general form there is no difference in the essential part of the theory, so we will assume that

$$\begin{aligned} Ls[\quad] &= \sum_n A_n \frac{d^n}{dt^n} \\ L[\quad] &= \sum_n B_n \frac{d^n}{dt^n} \end{aligned} \tag{3.7}$$

The previous equation takes the form

$$\sum A_n \frac{d^n}{dt^n} fs\,(t; \Delta) = \sum B_n \frac{d^n}{dt^n} f(t; \Delta) \tag{3.8}$$

or

$$\sum A_n \frac{d^n}{dt^n} \int_{-\infty}^{\infty} fs^*(p; \Delta) \exp\,(ipt)dp = \sum B_n \frac{d^n}{dt^n} \int_{-\infty}^{\infty} f^*(p; \Delta) \exp\,(ipt)dp \tag{3.9}$$

Usually, the functions we are treating are uniformly convergent and we may change the order of differentiation and integration. Therefore we have

$$\begin{aligned} &\int_{-\infty}^{\infty} \sum_n A_n\,(ip)^n\, fs^*\,(p; \Delta) \exp\,(ipt)\,dp \\ &\qquad = \int_{-\infty}^{\infty} \sum_n B_n\,(ip)^n\, f^*\,(p; \Delta) \exp\,(ipt)\,dp \end{aligned} \tag{3.10}$$

Since this relation holds without regard to t, we have

$$\sum_n A_n\,(ip)^n\, fs^*\,(p; \Delta) = \sum_n B_n\,(ip)^n\, f^*\,(p; \Delta)\,. \tag{3.11}$$

Hence we have

$$f^*(p; \Delta) = \{\sum_n A_n\,(ip)^n / \sum_n B_n\,(ip)^n\} \cdot fs^*\,(p; \Delta) \tag{3.12}$$

Using the notation $\mathfrak{S}s(p;\,\Delta)$ analogous to $\mathfrak{S}(p;\,\Delta)$ which was defined by (3.4), and also

$$\gamma(p) = -\arg\,\{\,\sum A_n\,(ip)^n\,/\,\sum B_n\,(ip)^n\} \tag{3.13}$$

we have

$$\mathfrak{S}(p;\Delta) = \gamma(p) + \mathfrak{S}s(p;\Delta) \tag{3.14}$$

If the seismographs used at $\Delta = \Delta_1$ and Δ_2 have the same characteristics, $\gamma(p)$, the phase lag caused by the instruments, cancel each other and we have a type of formula similar to (3.5), namely,

$$V(p) = p(\Delta_2 - \Delta_1)\,/\,\{\mathfrak{S}s(p;\,\Delta_2) - \mathfrak{S}s(p;\,\Delta_1)\}$$

$$\mathfrak{S}s(p;\Delta) = \arctan\left\{\int_{-\infty}^{\infty} fs(t;\,\Delta)\sin pt\,dt \Big/ \int_{-\infty}^{\infty} fs(t;\,\Delta)\cos pt\,dt\right\} + 2m\pi \tag{3.15}$$

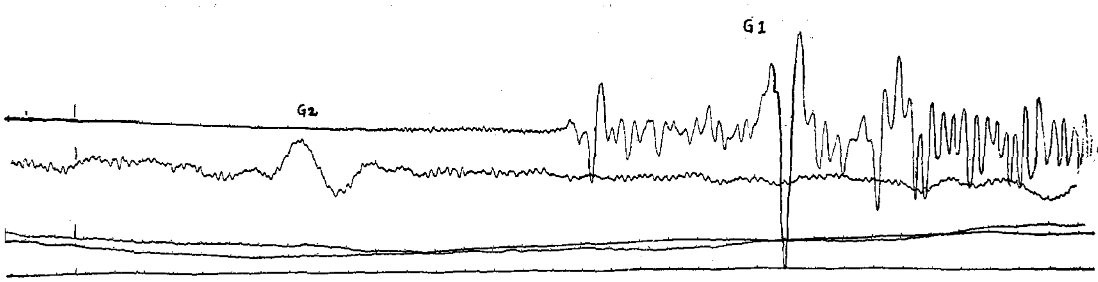


Fig. 8. Strain seismogram of Kamchatka earthquake. G1K implies G_1 phase of Kamchatka earthquake. (Courtesy of Professor H. Benioff.)

3.2. Phase and group velocity:

There is one thing which must be added to the foregoing theory. Since the inverse trigonometric function is a many-valued function, neither the expression (3.5) nor (3.15) can give the phase velocity uniquely. Corresponding to many values of n in (3.5) or m in (3.15), many curves are given, and without some other knowledge about the velocity of the medium we cannot decide which one of these is the true one. Unfortunately, group velocity cannot give any information about this ambiguity, because whatever value of n or m we may take, group velocity becomes always the same. This can easily be proved.

From (3.5) we have

$$\mathfrak{S}(p;\Delta_2) - \mathfrak{S}(p;\Delta_1) = p(\Delta_2 - \Delta_1)/V(p) = (\Delta_2 - \Delta_1)f \tag{3.16}$$

where $f = 2\pi/$wave length.

Differentiating by p,

$$\frac{d}{dp}\,[\mathfrak{S}(p;\,\Delta_2) - \mathfrak{S}(p;\,\Delta_1)] = (\Delta_2 - \Delta_1)/U\,. \tag{3.17}$$

The group velocity U is determined without regard to the values of integers involved in the expression (3.5).

3.3. Analysis of the seismograms of New Guinea and Kamchatka earthquakes:

We applied the theory described above to the analysis of the seismograms of the New Guinea and Kamchatka earthquakes. Seismograms were shown in figures 2 and 8. When we compare G_1 and G_3 observed by the same seismograph, the instrumental constants are exactly the same and we can use the formula (3.15). Dispersion curves obtained by this method are given in figure 12.1. We cannot decide which one of the curves in this figure is the true dispersion curve.[7] We also prepared the curves in figure 12.2 giving the relation between the velocity and the wave length. This type of curve is convenient for determining the group velocity, which is given by the intercept value of a tangent of the curve at any point.

As was described in section 1, the G wave shows slight dispersion, a fact which suggests that the curve group velocity versus wave length will approximately become a straight line. Figure 12.2 fits well with this idea, and, in the range of observation with fairly large amplitude, group velocity takes a nearly constant value of about 4.4 km/sec., which is in good agreement with the value given by H. Benioff.[8]

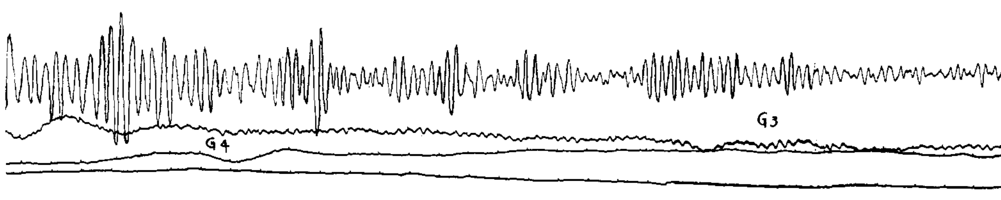


Fig. 8. (*Continued*)

We also used the combination of G_2 and G_4 and the result is given in figure 13. The similarity of curves in figures 12.1 and 13.1 is remarkable. The result obtained using the data of the Kamchatka earthquake is given in figure 14. The next step should be a determination of the distribution of shear velocity which will function as the wave guide of surface waves of the shear type and hold group velocity constant.

4. Wave guide of the G wave

Up to the present the wave channel that guides the G wave has not been identified. However, it seems that this phase corresponds to a Love wave traveling in the mantle and controlled by the gradient in shear velocity. Hence we may apply the theory of Love waves in a heterogeneous medium developed by T. Takahashi a few years ago.[9] Only by the application of his elegant method can we deduce the shear velocity without depending upon the tentative method of trial and error.

[7] We can determine the wave velocity uniquely if we have two observations spaced closely enough.

[8] H. Benioff and others, "Progress Report of the Seismological Laboratory," *Trans. Am. Geophys. Union*, 36, No. 4 (1955). He gives the value 4.38 km/sec., using the time of complete circuit of the earth for the G wave, 152.3 min.

[9] *Loc. cit.* (n. 4, *supra*).

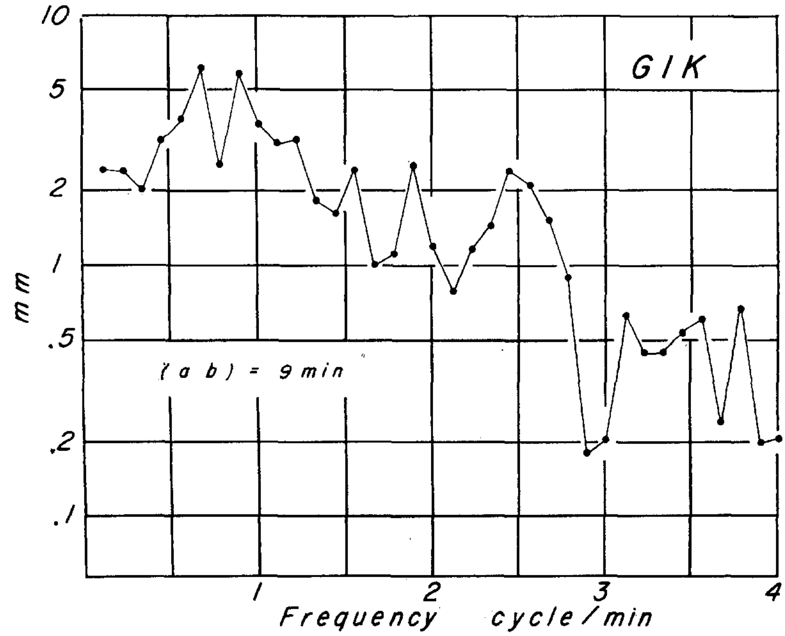


Fig. 9. Spectrum of G phases of Kamchatka earthquake.

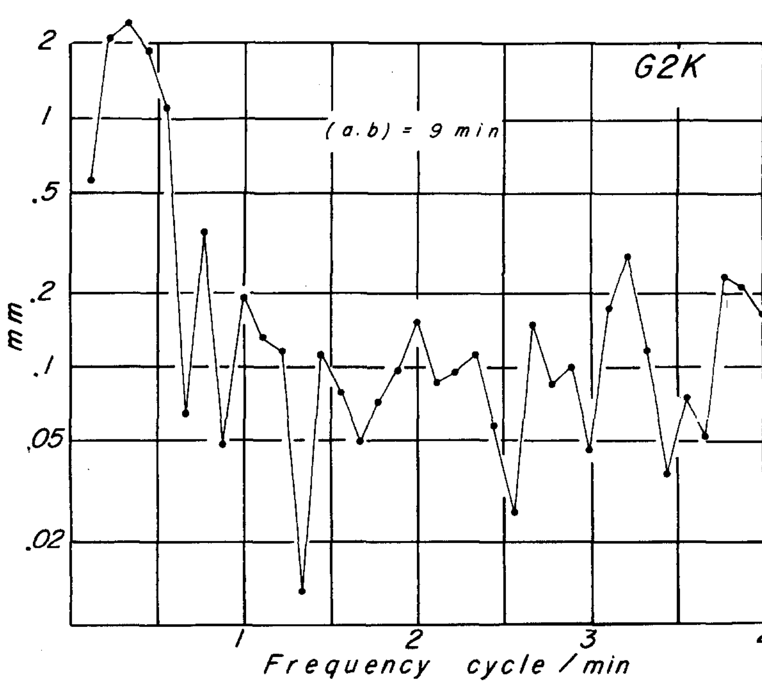
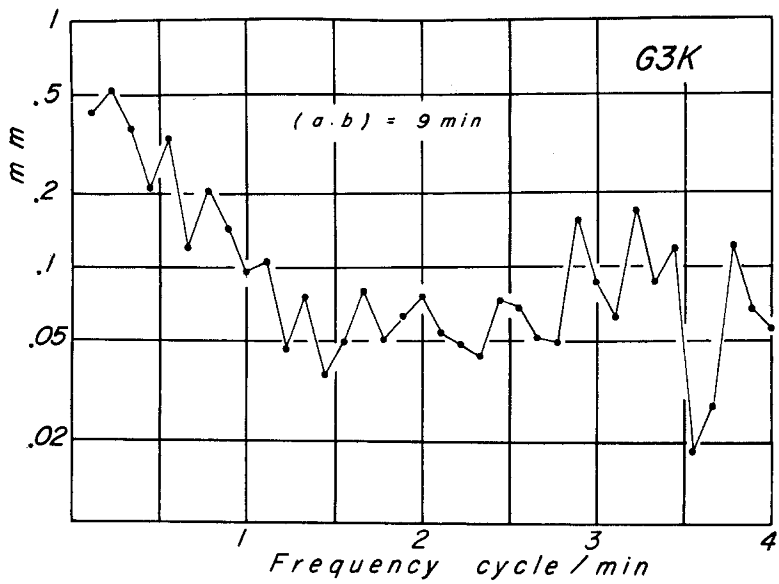


Fig. 9. (*Continued*)



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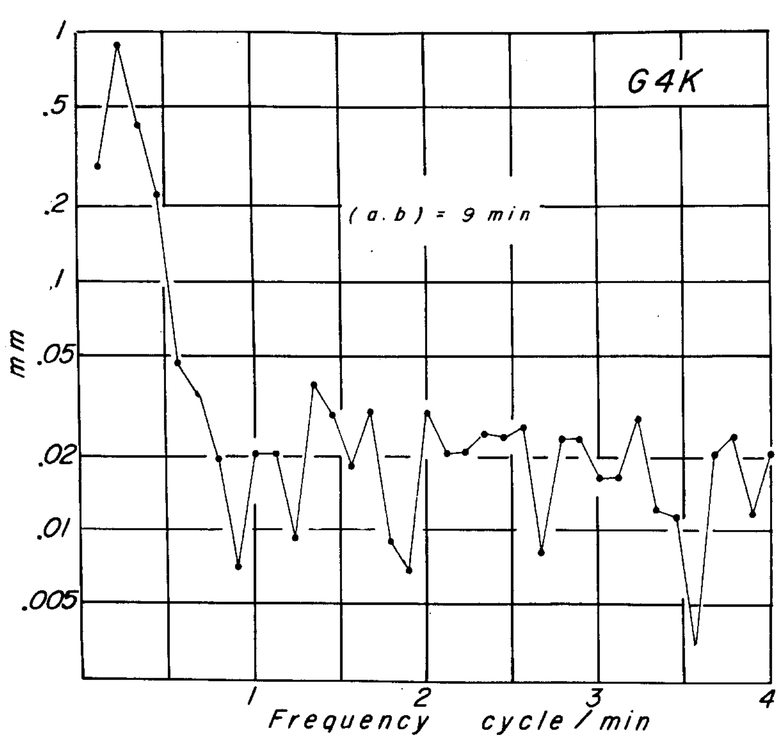

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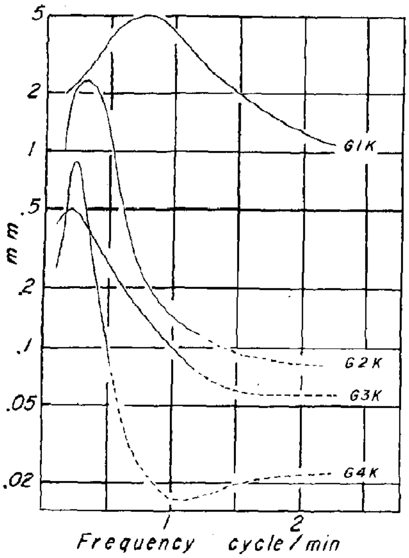

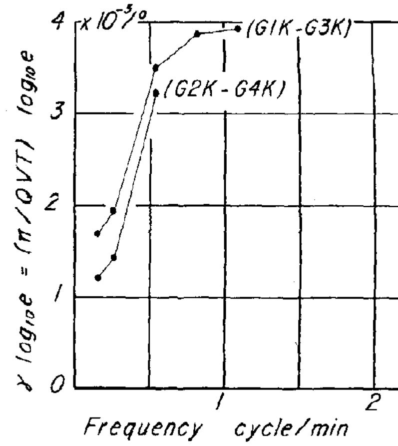

Fig. 10. (Left). Smoothed spectra of G phases of Kamchatka earthquake.

Fig. 11. (Right). The value of decrement γ in the expression (2.3). V: phase velocity. T: period.

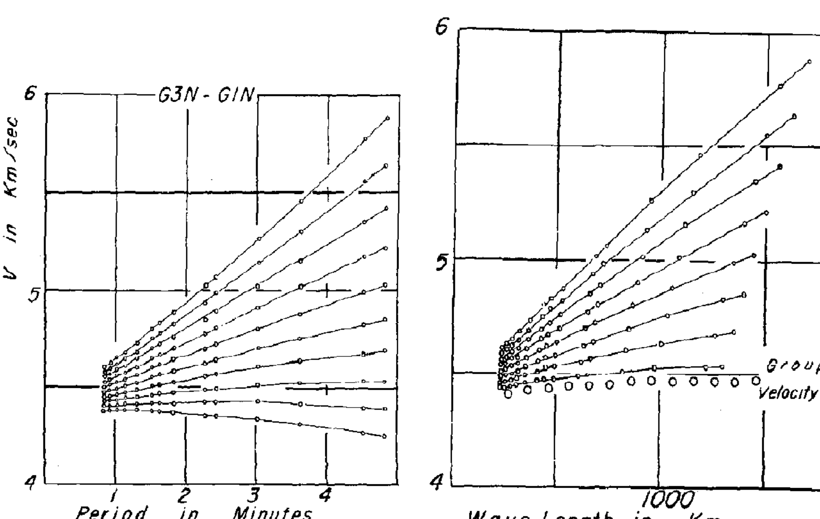

Fig. 12.1 (left) and 12.2 (right). Dispersion curve of G wave obtained using G_1 and G_3 phases of New Guinea earthquake.

According to his theory, if the dispersion curve of Love waves is given, the distribution of shear velocity is expressed by the following expression:[10]

$$z(V_s) = \frac{1.11}{4\pi} V_s \int_0^{T_s} \left\{ \frac{V_s^2}{V(T)^2} - 1 \right\}^{\frac{1}{2}} dT \tag{4.1}$$

where $V_s(z)$ is the shear velocity as a function of z, the depth from the free surface, V is the phase velocity of Love waves, and T_s is the period that satisfies the relation $V(T_s) = V_s$. Here the following two relations are assumed, namely, at the free surface

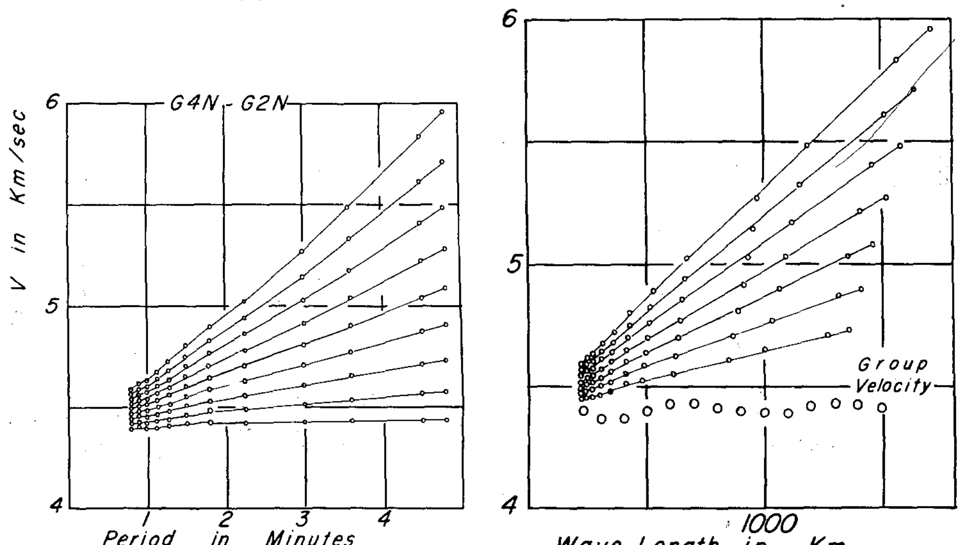

Fig. 13.1 (left) and 13.2 (right). Dispersion curve of G wave obtained using G_2 and G_4 phases of New Guinea earthquake.

$$\frac{d}{dz}\left\{ \mu(z) \sqrt{\frac{V^2}{V_s(z)^2} - 1} \right\} = 0 , \qquad (\mu\text{: rigidity}) \tag{4.2}$$

and

$$\frac{d}{dz} V_s(z) > 0 .$$

The first condition, which requires the medium to be nearly homogeneous at the free surface, can be removed, resulting in a modification in the formula (4.1). The following discussion, however, will prove that the foregoing condition works well and there is no need to modify Takahashi's theory.

As was pointed out first and was shown by the seismogram in figures 2 and 8, the G wave consists of only an oscillation or two and does not show dispersive properties. This was also confirmed by the theory in the previous section. Phase velocity determined by ($G_3 - G_1$) and ($G_4 - G_2$) proved to be approximately a linear func-

[10] The notation was changed a little.

tion of the wave length in a certain range of period, hence the group velocity was nearly constant. Therefore we have

$$U = V - L\frac{dV}{dL} = V_0 \text{ (constant)} \tag{4.3}$$

L = wave length; U = group velocity

From this expression we easily obtain the following relation:

$$V = V_0 + \kappa L = V_0/(1 - \kappa T) \qquad \kappa = \text{constant}. \tag{4.4}$$

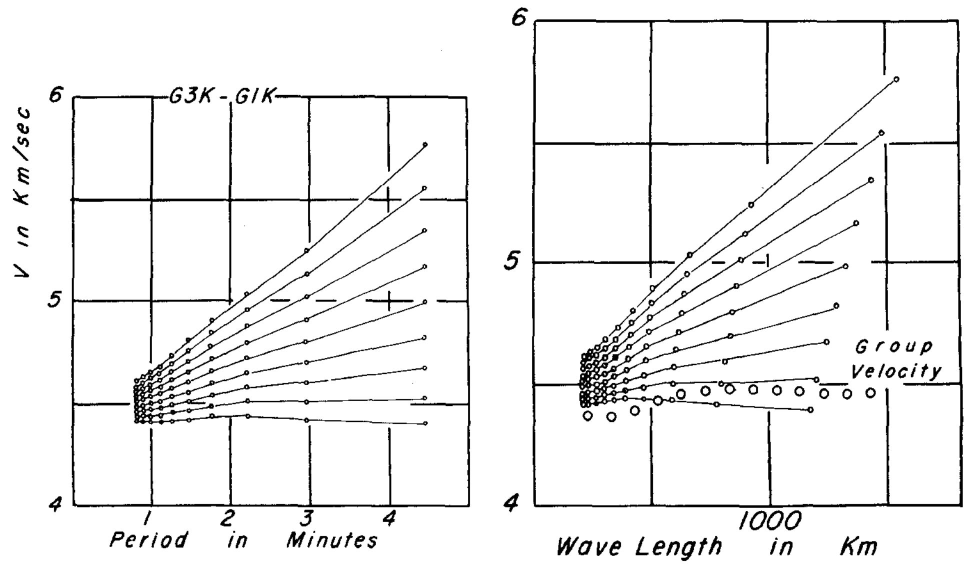

Fig. 14. Dispersion curve of G wave obtained using G_1 and G_3 phases of Kamchatka earthquake.

Since the last equation gives the relation between phase velocity and period, we can apply the method of T. Takahashi and obtain the distribution of shear velocity. The calculation is not difficult and the analytical expression of the depth z having a given shear velocity is

$$z(V_s) = K\left[\log\left\{\frac{V_s}{V_0} + \sqrt{\frac{V_s^2}{V_0^2} - 1}\right\}\right] \tag{4.5}$$

$$K = (1.11/4\pi)\cdot(V_0/\kappa)$$

Omitting a constant factor K the function

$$\log\left\{\frac{V_s}{V_0} + \sqrt{\frac{V_s^2}{V_0^2} - 1}\right\}$$

is shown in figure 15.

Takahashi's formula given above was derived from the next equation with a form similar to (4.1).

$$T(V) = \text{constant} \int_0^H \left\{ \frac{V^2}{V_s(z)^2} - 1 \right\}^{\frac{1}{2}} \frac{dz}{V} \tag{4.6}$$

where $V_s(H) = V$.

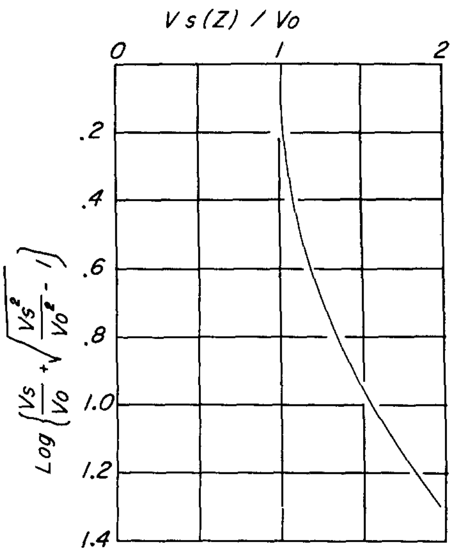

Fig. 15. Theoretically calculated distribution of the shear velocity which makes the group velocity constant.
$V_s = V_s(z)$ = shear velocity at the depth z.
$V_0 = V_s(0)$.

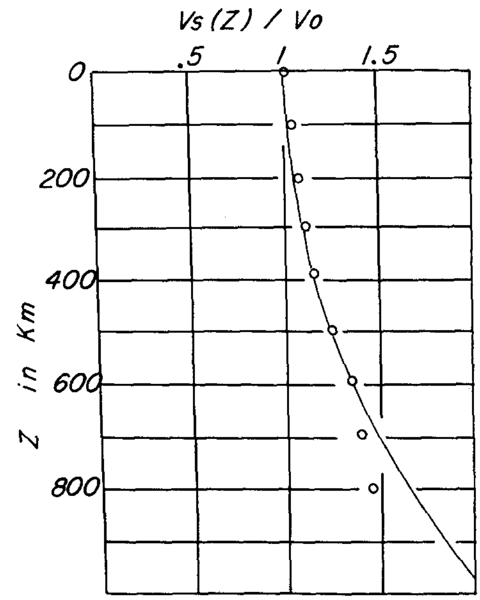

Fig. 16. Comparison of the theoretical distribution curve of the shear velocity (bold line) derived from the condition of constant group velocity and the values (circles) given by Jeffreys-Bullen table.

This formula shows that the period of a surface wave with a velocity $V = V_s(H)$ is determined by the distribution of shear velocity of the material at depths less than H.

If we introduce numerical values[11]

$$\begin{aligned} V_0 &= 4.4 \text{ km/sec.} \\ \kappa &= 5.3 \times 10^{-4} \text{ sec.}^{-1} \end{aligned} \tag{4.7}$$

into the expression (4.5), we have

$$\frac{1.11}{4\pi} \cdot \frac{V_0}{\kappa} = 733 \text{ km.}$$

[11] 4.4 km/sec, is confirmed from the travel time of the G wave and also from the intercept value of the tangent in figures 12.2 and 13.2. Cf. n. 8.
The value of κ was taken from the work of the present author. Y. Satô, "Analysis of Dispersed Surface Wave by Means of Fourier Transform, III: Analysis of Practical Seismogram of South Atlantic Earthquake," *Bull. Earthq. Res. Inst. Tokyo Univ.*, 34, p. 131 (1956).

Using this value as the unit length for the theoretical curve given above and also plotting the values of $V_s(z)/V_0$ derived from the distribution of shear velocity given by Jeffreys-Bullen, figure 16 was obtained.[12]

The distribution of a shear velocity at less than 600 km. depth is in good agreement with the theoretical curve obtained from the condition of constant group velocity. Corresponding to this depth, figure 16 gives $V_s(z)/V_0 = 1.32$. If we put this value and that of κ in (4.7) into the expression (4.4), we have $T = 7$ minutes, which is approximately the largest period of a wave with a significant amplitude.

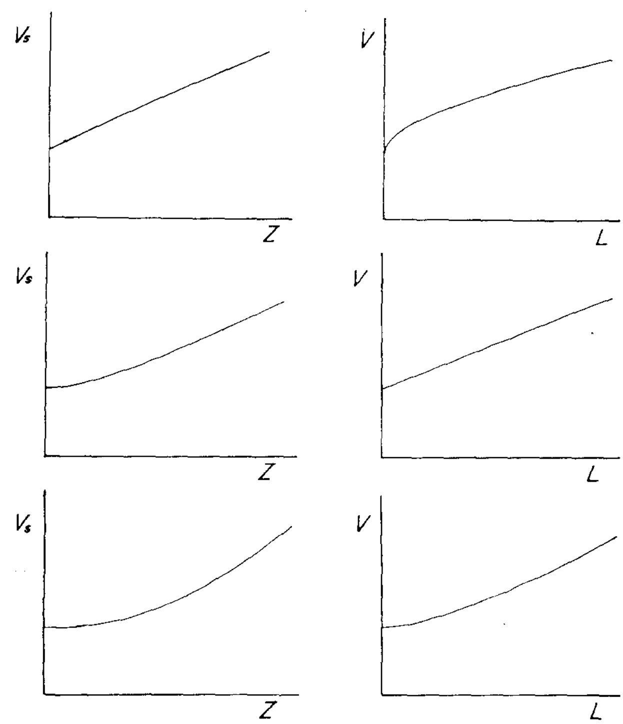

Fig. 17. Brief relation between the distribution of the shear velocity and the dispersion curve of surface waves. (Not to scale.)

4.1. Examination of the assumptions:

As was mentioned in section 4, we can remove the first condition of (4.2). However, the coincidence of the theoretical curve and the observed distribution reduced from the Jeffreys-Bullen table was remarkable, and at the present stage of approximation there will be no need for modification of the theory.

The second condition of continuously increasing shear velocity is hard to assume if we try to discuss the structure of the shallow part of the crust.

In figure 17 a brief sketch of the relation between the distribution of shear velocity and the dispersion curve is given. As we can see from these figures, the velocity distribution of the shallow part is sensitively affected by the shape of the

[12] In this computation V_0 was assumed to be 4.3 km/sec., which was obtained by extrapolating Jeffreys-Bullen's distribution. This is not exactly equal to the one in (4.7). This discrepancy must be a subject of our future study.

dispersion curve of short-period range. Therefore it is difficult for us to determine the shallow structure by means of a dispersion curve applying Takahashi's method.

The shortest wave which we dealt with in the previous section had a period of about 50 seconds, so we can say little about the part shallower than about 100 km. The foregoing theory will hold as an approximation between one hundred and several hundred kilometers in depth.

When we discuss a deeper part of the mantle using very long waves we should take into account the effect of the curvature of the earth.[13] This will be the next step of our study.

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[13] Cf. N. Jobert, "Effect de la courbure de la terre sur les ondes de Love," *Annales de Géophysique*, T. 11, No. 1 (1955).

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